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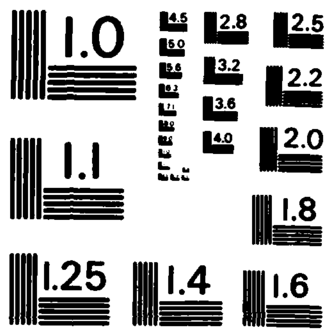

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EFFECT OF THE BASSET TERM ON PARTICLE RELAXATION BEHIND NORMAL SHOCK WAVES

Final Report

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Small particles and droplets encounter normal shocks in a variety of applications. The particle-shock interaction subjects the particles to large unsteady drag forces behind the shock front. In the present paper, an analysis has been made of the relative importance of the Basset history integral for particle displacement and velocity behind a normal shock wave. The effect of the Basset integral has been related to gas stagnation conditions and the local gas Mach number.

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In the present theoretical study it has been demonstrated that the particle velocity and displacement relative to the gas back of the shock is unaffected by the inclusion of the Basset term until the latter stages of particle relaxation. The effect of the Basset history integral, which results from diffusion of vorticity from the decelerating particle, has been shown to decrease the particle drag or increase the displacement of the particle back of the shock. The effect is magnified with increasing stagnation pressures and particle diameters but with decreasing gas stagnation temperatures.

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In the present theoretical study it has been demonstrated that the particle velocity and displacement relative to the gas back of the shock is unaffected by the inclusion of the Basset term until the latter stages of particle relaxation. The effect of the Basset history integral, which results from diffusion of vorticity from the decelerating particle, has been shown to decrease the particle drag or increase the displacement of the particle back of the shock. The effect is magnified with increasing stagnation pressures and particle diameters but with decreasing gas stagnation temperatures.

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I. INTRODUCTION

Liquid droplets, solid particles and agglomerates are prevalent in the atmosphere and in the by-product gases from combustion processes. These micron and submicron size particles are formed by condensation of supersaturated vapor and coagulation of existing aerosol. Common combustion devices responsible for the formation of large numbers of particle agglomerates are the internal combustion engine, power plants, jet engines and solid fueled rocket motors. Small particles are also introduced into wind tunnels to provide a seed for the measurement of fluid velocities with laser doppler techniques.

In many types of supersonic flows such as wind tunnel testing, jet and rocket engine plumes and high speed flight, the small particles and droplets encounter both normal and oblique shocks. The resulting particle-shock-interactions subject the particles to sudden large drag forces as the particles decelerate and project ahead of the fluid moving behind the shock front. In these cases an accurate description of the drag force on the particle is necessary to predict its trajectory. Specific problems of interest to the Air Force are the impingement of water droplets and ice crystals on supersonic airfoils [1], particle sampling with supersonic probes in jet and rocket motor plumes [2] and laser velocimetry measurements near shock fronts [3,4].

In the present study, the analysis of particle relaxation is restricted to the interaction of particles with normal shocks, specifically, normal shocks in isentropic supersonic flows. The purpose of the present

analysis is to determine the relative importance of the "Basset history integral," which results from diffusion of vorticity from the particle, to the particle velocity and displacement as it relaxes behind the shock front [5]. In particular, it is of interest to relate the magnitude of the effect of the Basset term to gas stagnation conditions and the local Mach number.

The Basset term has been neglected in particle-shock interactions [1,4] but calculations of particle trajectories in plasma jets indicate that it must be included in certain types of accelerated flows [6]. In the present case, the particles relax and decelerate relative to the gas back of the shock front. Although experimental evidence indicates that the particle drag decreases for a decelerating particle if the initial particle Reynolds number is large [7,8], the Basset term does not appear to be important for particle deceleration if $Re < 10$ [9] or if the particle decelerates followed by rapid acceleration such that the particle Reynolds number is always large [10]. Clearly, more theoretical and experimental work is necessary to clarify the issue.

II. OBJECTIVES

The objectives of the present study are to numerically compute the relative importance of the Basset term for particle relaxation behind a normal shock wave and to relate its effect to nozzle stagnation conditions and local gas Mach number. The specific objectives are:

(1) Define particle parameters behind a normal shock.

(2) Relate particle parameters and particle displacement, with and without the Basset term, to nozzle stagnation conditions and gas Mach number.

(3) Provide plots of particle displacement and velocity on contours of constant particle size, nozzle stagnation conditions and gas Mach number illustrating the importance of the Basset term for a wide range of expected nozzle operating conditions.

(4) Establish criteria for the neglect of the Basset term in particle-shock interactions.

III. ISENTROPIC FLOW

In the present study it is assumed that particles are in equilibrium with an isentropic supersonic gas prior to the particle-shock interaction. In this case all gas properties near the shock front are expressed in terms of gas stagnation conditions and the local gas Mach number. The same configuration is also easily obtained with a converging-diverging channel which is the basic aerodynamic element used to obtain prescribed supersonic flows in laboratory applications.

If the nozzle is supplied with gas at high pressures and temperatures (stagnation conditions) at the inlet and if the exhaust pressure is sufficiently low, sonic conditions exist in the throat and the gas Mach number at any position along the axis of the nozzle is determined by the ratio of the local cross sectional area to that of the throat. The same basic configuration also exists in the nozzle of a solid fuel rocket motor.

If the nozzle is designed to function without significant separation along the inside walls and we assume a perfect gas with constant specific heats, the flow is assumed to be isentropic and the gas properties are related to stagnation conditions by the following expressions [5],

$$\frac{T_o}{T_1} = 1 + \left(\frac{\gamma-1}{2}\right) M_1^2 \tag{1}$$

and

$$\frac{\rho_o}{\rho_1} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_1^2\right]^{\frac{1}{\gamma-1}} . \tag{2}$$

Stationary test objects such as airfoils or probes placed in the supersonic region of the nozzle flow will create discontinuities in the flow field. These shock waves may be normal or oblique to the direction of flow. Assuming a thermally and calorically perfect gas and restricting the discussion to normal shocks, the ratio of gas properties across the shock wave in terms of the gas Mach number ahead of the shock are

$$\frac{\rho_1}{\rho_2} = \frac{v_2}{v_1} = \frac{(\gamma-1)M_1^2 + 2}{(\gamma+1)M_1^2} \tag{3}$$

and

$$\frac{T_2}{T_1} = \frac{\left[1 + \left(\frac{\gamma-1}{2}\right) M_1^2\right]\left[\left(\frac{2\gamma}{\gamma-1}\right) M_1^2 - 1\right]}{\frac{(\gamma+1)^2}{2(\gamma-1)} M_1^2} . \tag{4}$$

Equations (1)-(4) will be used in the discussion that follows and refer to those gas properties shown in fig. 1.

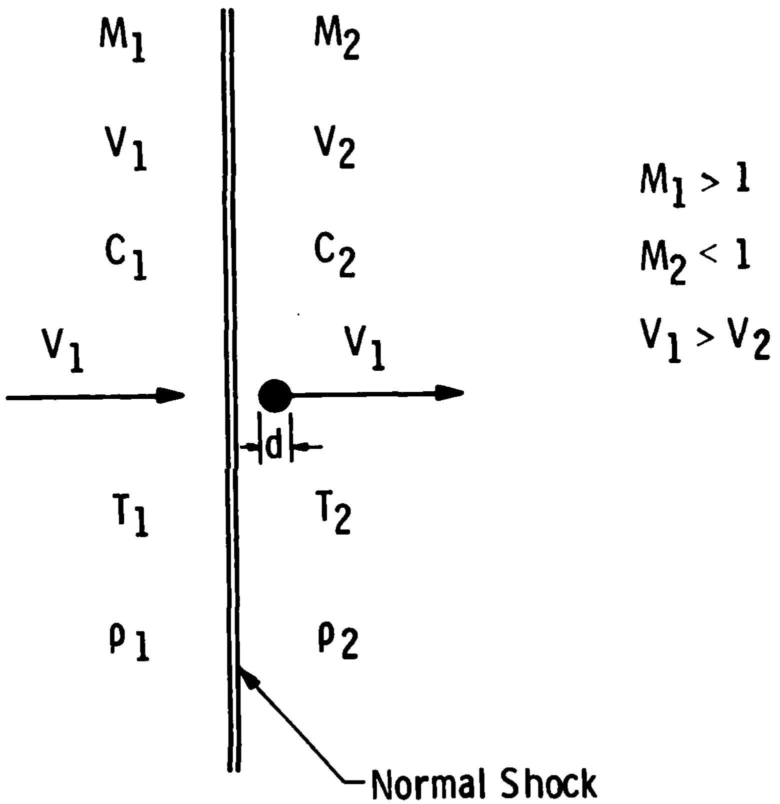


FIG. 1 – PARTICLE MOVING BEHIND NORMAL SHOCK WAVE

IV. PARTICLE PARAMETERS AND STAGNATION CONDITIONS

When a particle encounters a shock front as indicated in fig. 1, it projects ahead of the carrier gas moving behind the discontinuity because of its inertia and the sharp decrease in gas velocity. This phenomenon subjects the particle to a large unsteady drag force behind the shock wave and the particle decelerates and relaxes relative to the carrier gas.

To predict the particle trajectory behind the shock it is necessary to define three dimensionless groups as discussed below. These particle parameters are defined in terms of the gas stagnation conditions of the nozzle, particle properties and the gas Mach number M_1 upstream of the normal shock.

1. Kundsen Number

The particle Knudsen number is defined as the ratio of the mean free path of the gas to the particle diameter. From kinetic theory [12] one obtains for the stagnation Knudsen number

$$Kn_o = \left(\frac{\pi\gamma}{2}\right)^{1/2} \frac{\mu_o}{c_o \rho_o d} \tag{5}$$

where $c_o = (\gamma R T_o)^{1/2}$ is the speed of sound in the stagnation reservoir.

Introducing Sutherland's formula for viscosity

$$\mu = \frac{bT^{3/2}}{T+T_\theta} \tag{6}$$

where

$$b = \mu_r \left(\frac{1}{T_r}\right)^{1/2} \left(1 + \frac{T_\theta}{T_r}\right) , \tag{7}$$

such that

$$\frac{\mu_o}{\mu_r} = \left(\frac{T_o}{T_r}\right)^{1/2} \left[\frac{1 + T_\theta/T_r}{1 + T_\theta/T_o}\right] , \tag{8}$$

$\mu_r = \mu(T_r)$ is a reference viscosity and T_θ is Sutherland's constant [13], the stagnation Knudsen number can be written in the form

$$Kn_o = \left(\frac{\pi}{2}\right)^{1/2} \frac{k}{\rho_o d} \left(\frac{1}{1 + T_\theta/T_o}\right) . \tag{9}$$

Here, $k = b/R^{1/2}$ and R is the specific gas constant.

Introducing local properties into Eq. (9) by writing all gas properties in the form

$$\rho_o = \left(\frac{\rho_1}{\rho_2}\right) \left(\frac{\rho_o}{\rho_1}\right) \rho_2 \tag{10}$$

where the ratios of gas properties are given by Eqs. (1)-(4), one obtains a value for the Knudsen number back of the shock front

$$Kn_2 \cong Kn_o \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{\rho_o}{\rho_1}\right) . \tag{11}$$

2. Reynolds Number

The particle Reynolds number which represents the ratio of inertial to viscous forces acting on the particle is defined in terms of the particle diameter and its local velocity relative to the ambient gas. The local Reynolds number is a maximum behind the shock front and subsequently approaches zero as the particle equilibrates to the gas velocity. Thus, immediately behind the shock wave one obtains

$$Re_2 = \frac{\rho_2(v_1-v_2)d}{\mu_2} \quad . \tag{12}$$

It is now convenient to define a particle Mach number immediately behind the shock front which represents the ratio of the relative velocity of the particle with respect to the ambient gas to the local speed of sound. Thus

$$Mp_2 = \frac{v_1-v_2}{c_2} = \left(1 - \frac{v_2}{v_1}\right)\left(\frac{T_1}{T_2}\right)^{1/2} M_1 \quad . \tag{13}$$

From kinetic theory Mp_2 is not independent of Re_2 and Kn_2 since $Mp_2 = Kn_2 Re_2 (2/\pi\gamma)^{1/2}$ [12]. Therefore, one obtains the local particle Reynolds number back of the shock front in the form,

$$Re_2 = \left(\frac{\pi\gamma}{2}\right)^{1/2}\left(\frac{1}{Kn_o}\right)\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\rho_1}{\rho_o}\right) f(M_1) \tag{14}$$

where $f(M_1) = Mp_2$ and

$$f(M_1) = \frac{\left(\frac{2}{\gamma-1}\right)^{1/2}(M_1^2 - 1)}{\left[1 + \left(\frac{\gamma-1}{2}\right)M_1^2\right]^{1/2}\left[\left(\frac{2\gamma}{\gamma-1}\right)M_1^2 - 1\right]^{1/2}} . \quad (15)$$

3. Density Ratio

The remaining dimensionless group which is necessary to compute the particle trajectory back of the shock front is the ratio of local gas density to particle density. Thus, one obtains

$$\frac{\rho_2}{\rho_p} = \left(\frac{\rho_o}{\rho_p}\right)\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\rho_1}{\rho_o}\right) \quad (16)$$

where ρ_p is the particle density and the remaining gas density ratios are determined from Eqs. (1)-(4).

4. Tabulated Stagnation Conditions

The stagnation conditions have been computed for a useful range of reservoir pressures, temperatures and particle diameters. For convenience, the computations have been restricted to air and particle densities equal to that of water or $\rho_p = 1$ gm/cm^3. Values of the ratio of stagnation density to particle density ρ_o/ρ_p are tabulated in table 1 of sec. 1 of the appendix. In table 1 four values of stagnation pressure $P_o = 14.7$, 50, 100, 500 psi were chosen along with four values of stagnation temperature $T_o = 300$, 500, 1000 and 3500°K. The particle Knudsen number Kn_o was also computed for the stagnation conditions listed above and for four particle diameters of d = 0.1, 1, 10 and 100 μm and these values are listed in tables 2-4 of the appendix. In tables 1-4 of the appendix it

was assumed that the reference viscosity of air was $\mu_r = 1.71 \times 10^{-4}$ gm/cm-sec at a reference temperature of $T_r = 273.2°K$. In addition, Sutherland's constant of $T_\theta = 111.3°K$ for air and a specific gas constant of $R = 2.88 \times 10^6$ cm^2/sec^2-°K were introduced into Eq. (7) to compute a value of $k = b/R^{1/2} = 0.859 \times 10^{-8}$ gm/cm^2 [12,13,14,15]. The information tabulated in tables 1-4 along with values for the local Mach number and ratio of specific heats for the gas were used to compute values of the particle Reynolds number Re_2, Knudsen number Kn_2 and density ratio ρ_2/ρ_p back of the shock from Eqs. 11, 12 and 16.

V. EQUATION OF MOTION

Restricting the analysis to the rectilinear acceleration of a rigid sphere, the equation of motion including the effect of large particle Reynolds number can be written in the form [16,17,18],

$$-F_D = F_V + F_M + F_B \tag{17}$$

where F_D is the total drag on the particle, F_V is the viscous drag, F_M is the added mass term and F_B is the Basset term. The terms in Eq. (18) are defined as

$$F_D = m_p \frac{dv}{dt} \quad , \tag{18}$$

$$F_V = \frac{\rho}{2} C_D A_p u^2 \quad , \tag{19}$$

$$F_M = \frac{\Delta_A}{2} u\rho \frac{du}{dt} \quad , \tag{20}$$

and

$$F_B = \frac{3\Delta_H}{2} d^2 \sqrt{\pi\rho\mu} \int_o^t \frac{\dot{u}(s)ds}{(t-s)^{1/2}} \quad . \tag{21}$$

In Eqs. (18) to (21), $u = v - v_2$ where v is the particle velocity and Δ_A, Δ_H are empirical coefficients to account for differences from creeping flow.

In dimensionless form, assuming that the particle density is much greater than the ambient gas density, Eq. (18) becomes

$$\frac{2\beta}{9}\dot{Re} = -\frac{A_D}{24} - \frac{\Delta_H}{\sqrt{\pi}}\int_o^\tau \frac{\dot{Re}(\sigma)\,d\sigma}{(\tau-\sigma)^{\frac{1}{2}}} \quad . \tag{22}$$

Here, $Re = \frac{du}{\nu}$, $\dot{Re} = \frac{dRe}{d\tau}$, $\tau = \frac{4\nu t}{d^2}$, $\sigma = \frac{4\nu s}{d^2}$, $A_D = C_D Re^2$ and $\beta = \frac{\rho_p}{\rho}$. Since the drag coefficient $C_D = C_D(Re,Kn)$ is a complex function [18] as shown in fig. 2 and since Δ_H is defined as

$$\Delta_H = 0.48 + \frac{0.52\, M_A^3}{(1+M_A)^3} \tag{23}$$

where $M_A = \frac{d}{u^2}\left(\frac{du}{dt}\right) = \left(\frac{4\dot{Re}}{Re^2}\right)$ is the acceleration modulus [19], Eq. (22) must be solved numerically subject to the boundary conditions:

$$\begin{aligned} &\tau = 0, \qquad Re = Re_2 \\ &\tau > 0, \qquad Kn = Kn_2, \qquad \beta = \rho_p/\rho_2 \end{aligned} \tag{24}$$

where Re_2 is the maximum value of the particle Reynolds given by Eq. (12).

The displacement of the particle relative to the fluid can be determined numerically by the expression

$$x/d = \frac{1}{4}\int_o^\tau Re\, d\tau \tag{25}$$

where $x/d = \frac{(x_p - x_f)}{d}$ and x_p,x_f are the particle and fluid displacement back of the shock front, respectively. An additional quantity E was also defined to represent the defect in particle displacement, with and without

the Basset term, or

$$E = \frac{xb/d - x/d}{xb/d} \tag{26}$$

where xb/d and x/d are the relative particle displacements including and excluding the Basset term respectively.

The drag coefficient C_D which appears in Eq. (22) and is illustrated in fig. 2 is the algebraic expression proposed by Crowe [19]. Thus,

$$C_D = (C_{D1} - 2)\exp\left[-3.07\,\gamma^{1/2}(Mp/Re)g(Re)\right]$$

$$+\left[h(Mp)/(\gamma^{1/2}Mp)\right]\exp\left[-Re/(2Mp)\right] + 2 \quad , \tag{27}$$

where the drag coefficient in incompressible continuum flow is [14,15]

$$C_{D1} = (24/Re)(1 + 0.158\,Re^{2/3}) \quad . \tag{28}$$

The remaining terms are

$$\log_{10}g(Re) = 1.25[1 + \tanh(0.77\,\log_{10}Re - 1.92)] \tag{29}$$

and

$$h(Mp) = (2.3 + 1.7(Tp/Tg)^{\frac{1}{2}}) - 2.3\tanh(1.17\,\log_{10}Mp)\,. \tag{30}$$

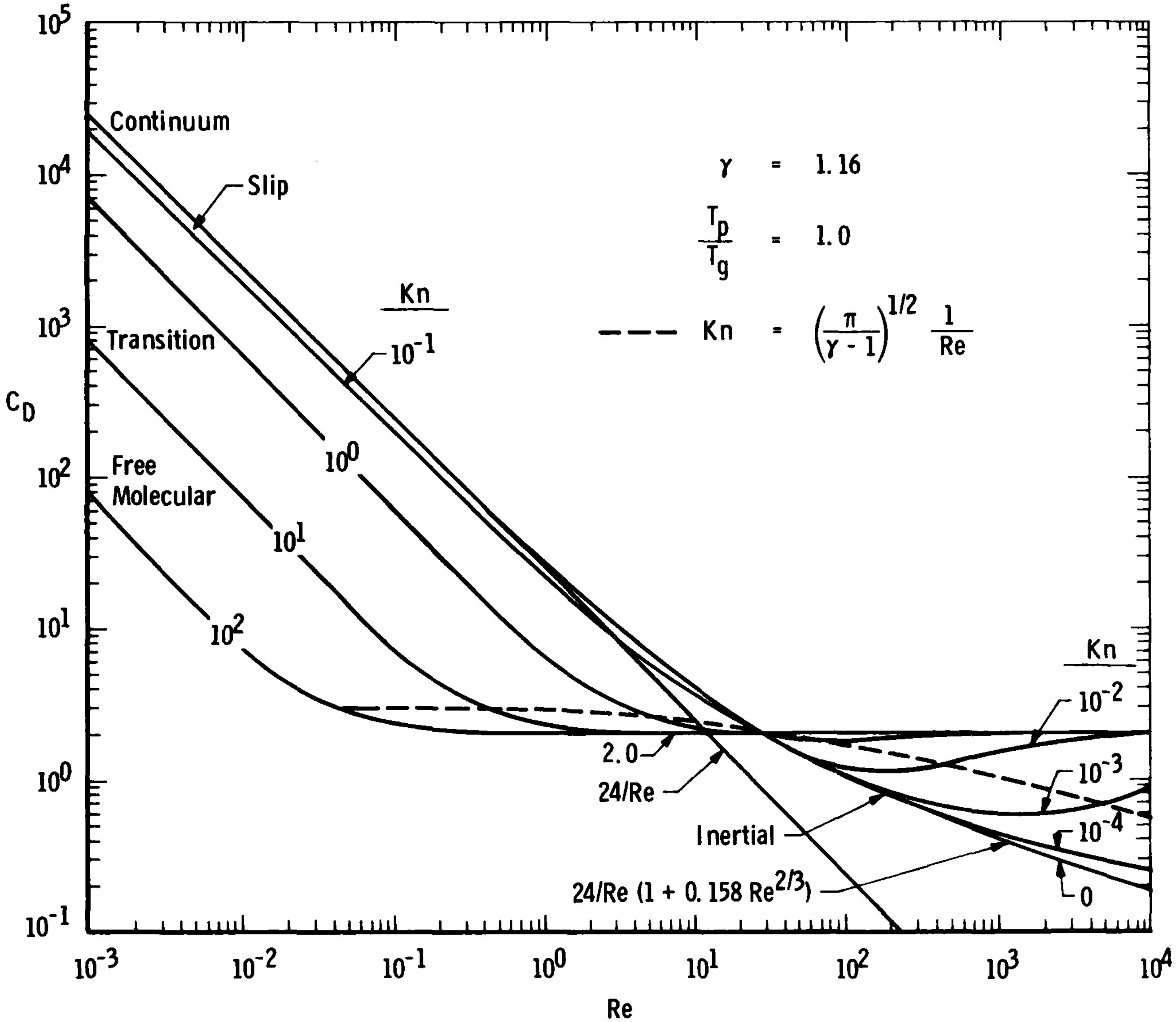


FIG. 2 — PARTICLE DRAG COEFFICIENT

VI. NUMERICAL METHODS

The equation of motion for the particle, Eq. (22), was solved numerically for a range of normalized times $0 \leqslant \tau \leqslant 1.2 \times 10^4$. Values of particle Reynolds number Re and relative particle displacement x/d were tabulated, with and without the Basset term, for a variety of anticipated stagnation conditions. In addition, the defect in relative particle displacement E was computed for each set of initial conditions. These tabulated results are shown in sec. 2 of the appendix.

The numerical procedures used to solve Eq. (22) were a fourth order Runge-Kutta if the Basset term was excluded from the equation of motion and a modified Euler, predictor-corrector technique for the full equation including the Basset term [20]. In the latter case the corrector was applied three times to improve convergence. Moreover, for each step forward in normalized time $\Delta\tau = 0.1$, the Basset integral was numerically evaluated with the trapezoidal rule for the first 160 steps followed by Simpson's rule with a variable (increasing) step size to reduce computer time and then finished with the trapezoidal rule to complete the integration.

A preliminary numerical computation was performed to compare the numerical results with an exact solution. This identified potential errors and problems with the accuracy of the method. The exact solution used was the case of creeping flow (small initial particle Re) and a density ratio $\rho/\rho_p = 2$ [18]. After considerable numerical experimentation it was found that the error in the particle displacement defect E

was $< 1.7\%$ and decreasing at $\tau = 10^3$ for a step size $\Delta\tau = 0.1$. A complete listing of the numerical code is shown in sec. 3 of the appendix.

VII. NUMERICAL RESULTS

Numerical computations were performed for a variety of test cases and the numerical results are tabulated in sec. 2 of the appendix.

1. Particle Relaxation

Figures 3, 4 and 5 are graphical illustrations of the tabulated data for the relaxation behind the shock front of a 10 μm particle of density $\rho_p = 1$ gm/cm^3 traveling in air at a Mach number of 2. The stagnation conditions for this case are a particle Knudsen number $Kn_o = 9.8 \times 10^{-4}$ and a ratio of gas-to-particle density $\rho_o/\rho_p = 8 \times 10^{-3}$. The initial particle Reynolds number back of the shock is $Re_2 = 894$.

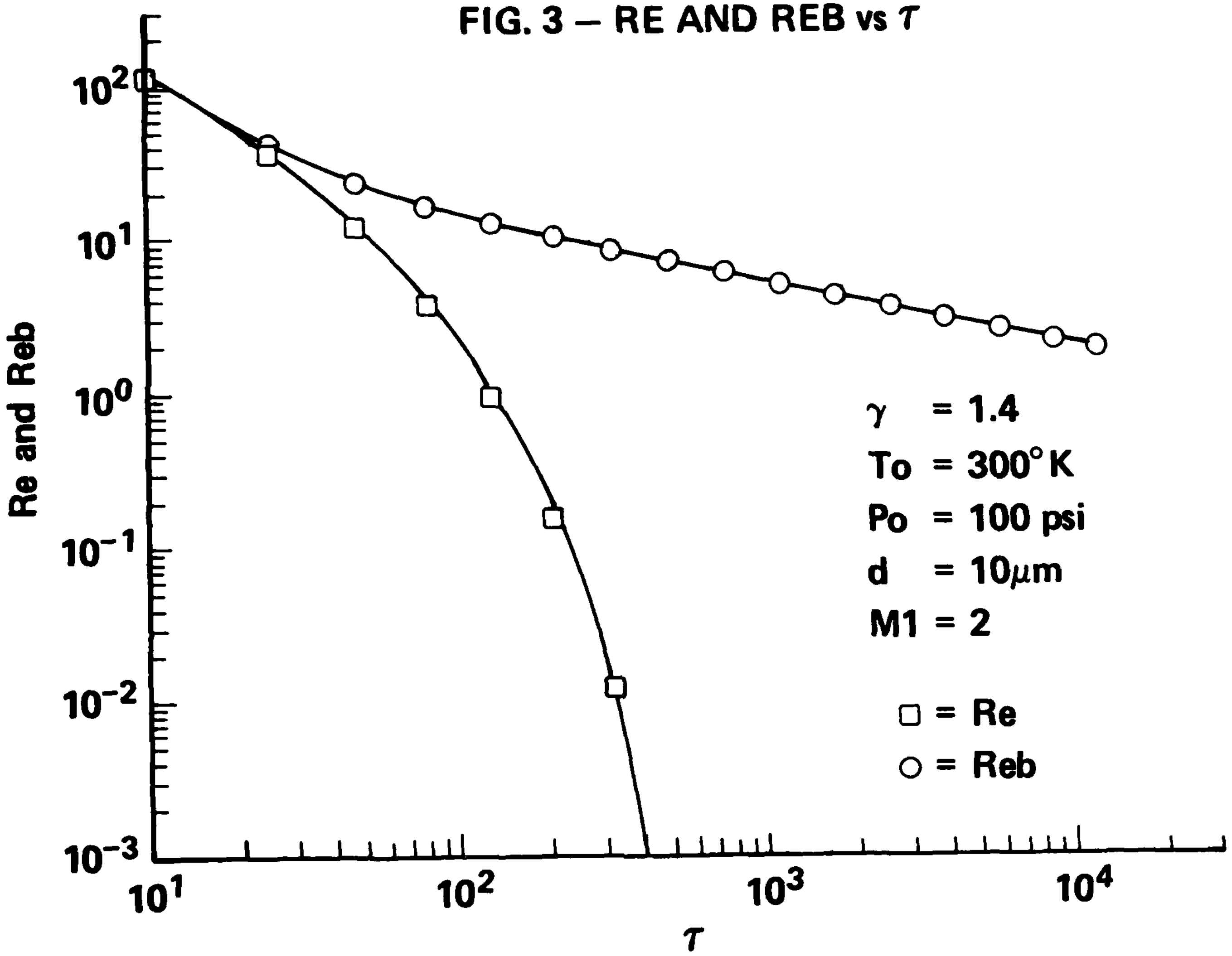
As illustrated in fig. 3, the particle Reynolds number including the Basset term Reb does not deviate significantly from the particle Reynolds number excluding the Basset term Re until Re ~ 10 or until Re is reduced to roughly 1% of its initial value. Moreover, the particle Reynolds number including the Basset term Reb slowly decreases but sustains a value of Reb ~ 1 for large values of normalized time $\tau >> 10^4$.

Figure 4 illustrates the particle relaxation distance relative to the gas back of the shock. The relaxation distance xb/d including the Basset term is roughly a factor of ten larger that the particle relaxation distance x/d excluding the Basset term at a normalized time of $\tau \cong 10^3$. These results are also reflected in fig. 5 which illustrates the defect in the particle relaxation distance E defined by Eq. (26).

2. Effect of Mach Number

Figure 6 represents the defect in particle relaxation distance at

FIG. 3 — RE AND REB vs τ



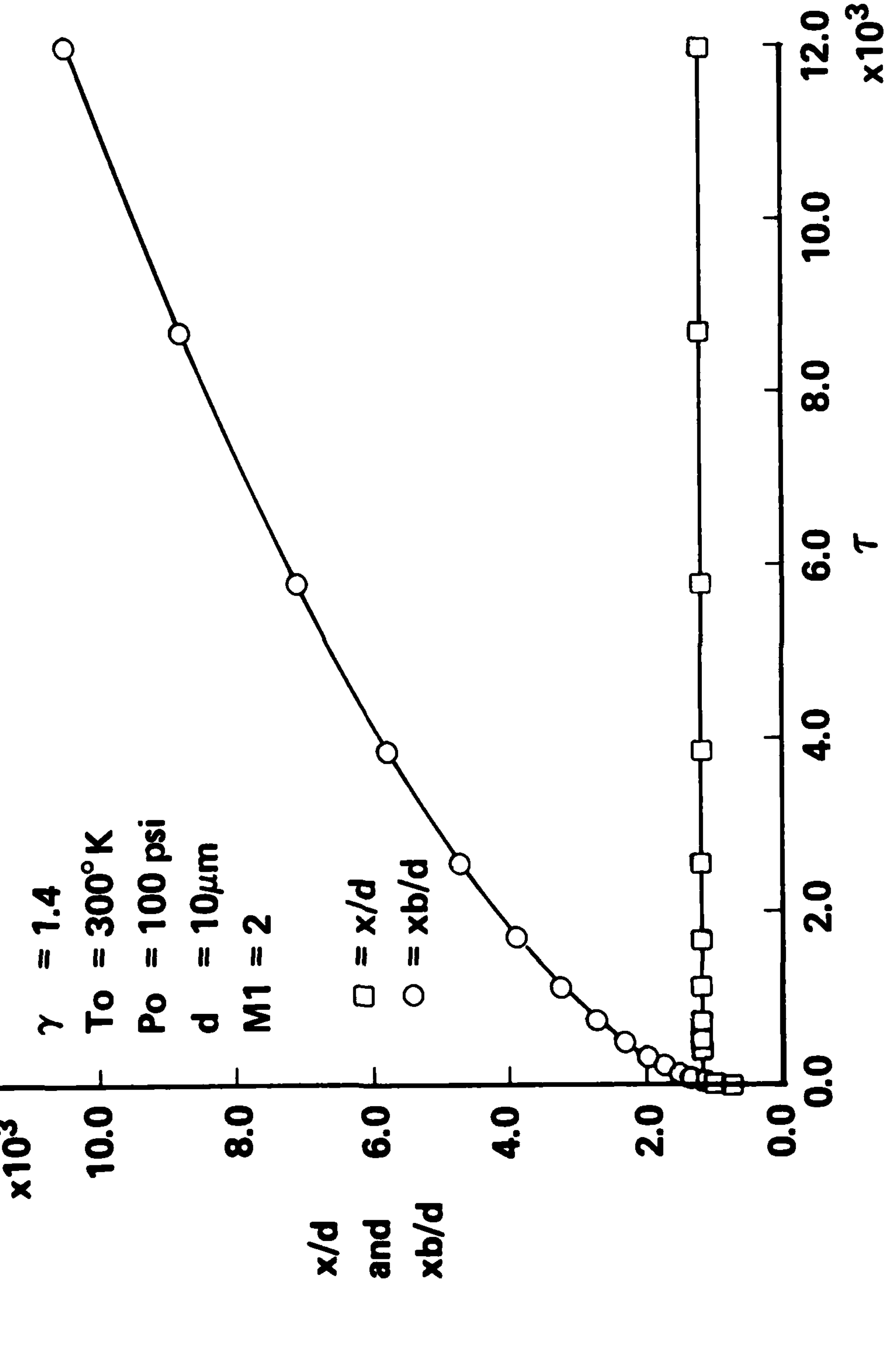


FIG. 4 — x/d and xb/d vs τ

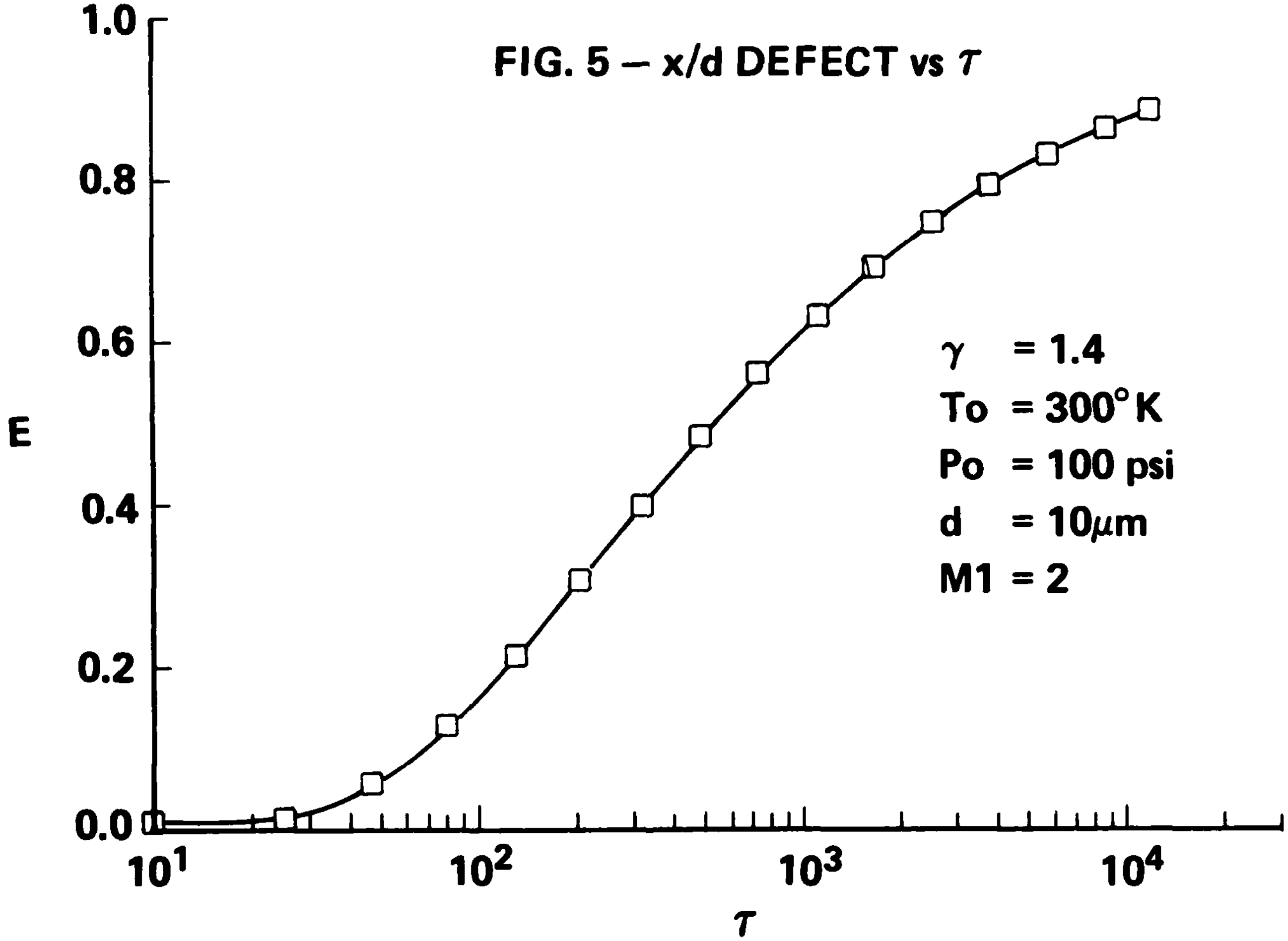


FIG. 5 — x/d DEFECT vs τ

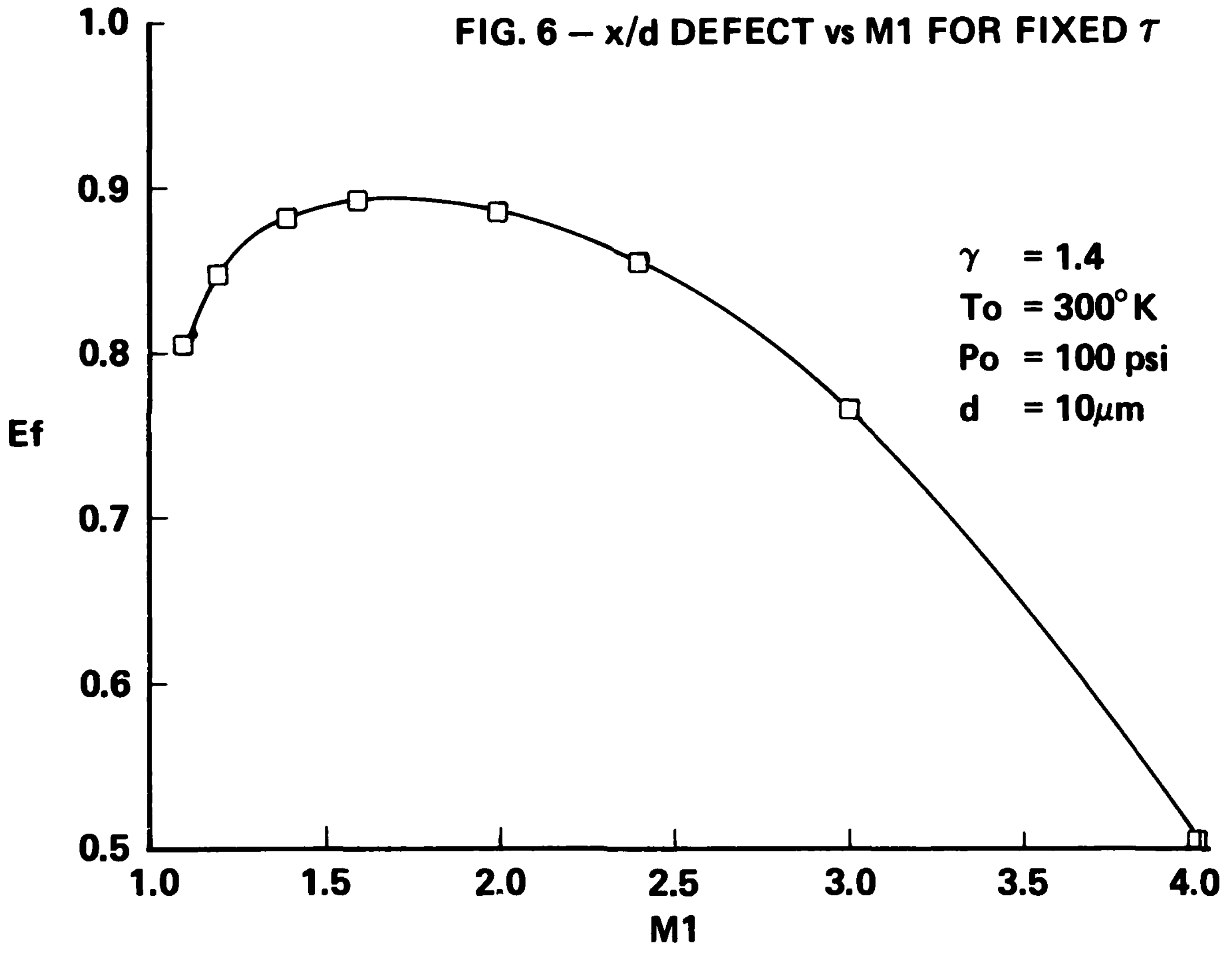


FIG. 6 – x/d DEFECT vs M1 FOR FIXED τ

a normalized time $\tau = 1.2 \times 10^4$. Previous work [14,15] indicates a maximum particle Reynolds number Re_2 immediately back of the shock front at a gas Mach number M_1 of roughly 2. Since the effect of the Basset term is magnified for larger initial particle Reynolds numbers and, in general, experimental measurements indicate a substantial reduction in particle drag coefficients at larger particle Reynolds numbers [8], a peak in the relaxation defect exists at a gas Mach number $M_1 \sim 1.75$.

3. Effect of γ

The defect in particle displacement was determined for two values of the ratio of specific heats $\gamma = 1.2$ and 1.4. As indicated in fig. 7, there is little difference between the curves at a gas Mach number of 2. However, it is expected that this difference would increase substantially at larger gas Mach numbers since a larger difference in initial particle Reynolds numbers for two values of γ does exist at larger values of M_1 [14,15]

4. Effect of Particle Diameter

The defect in the particle relaxation distance E is plotted as a function of normalized time τ for four particle diameters in fig. 8. Here as in previous discussions, larger particle diameters correspond to larger values of Re_2, the initial particle Reynolds number back of the shock. Thus, E increases with larger particle sizes for fixed normalized times τ.

5. Effect of Stagnation Pressure

The effect of increasing the stagnation pressure is illustrated in fig. 9. Since larger stagnation pressures correspond to larger stagnation

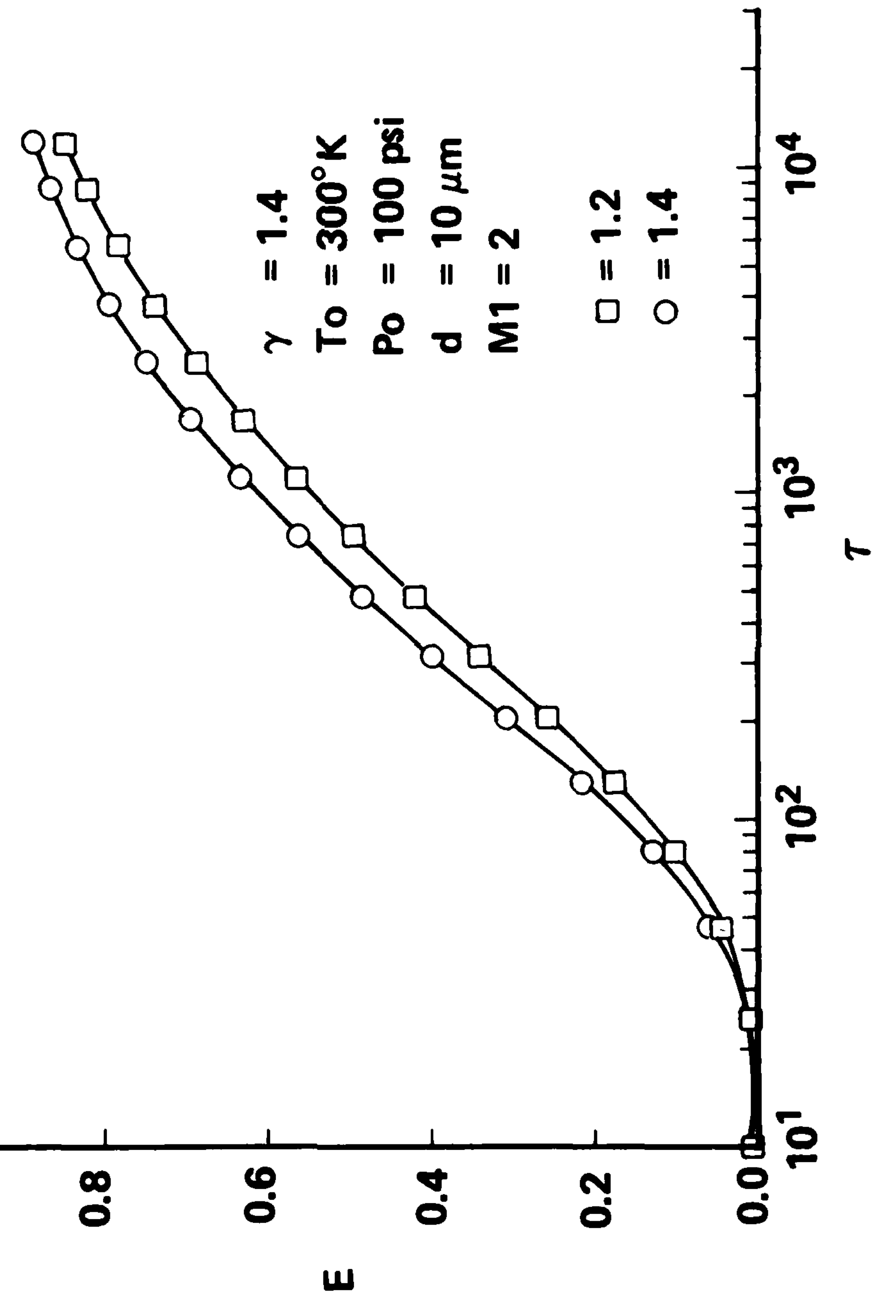


FIG. 7 — x/d DEFECT vs τ

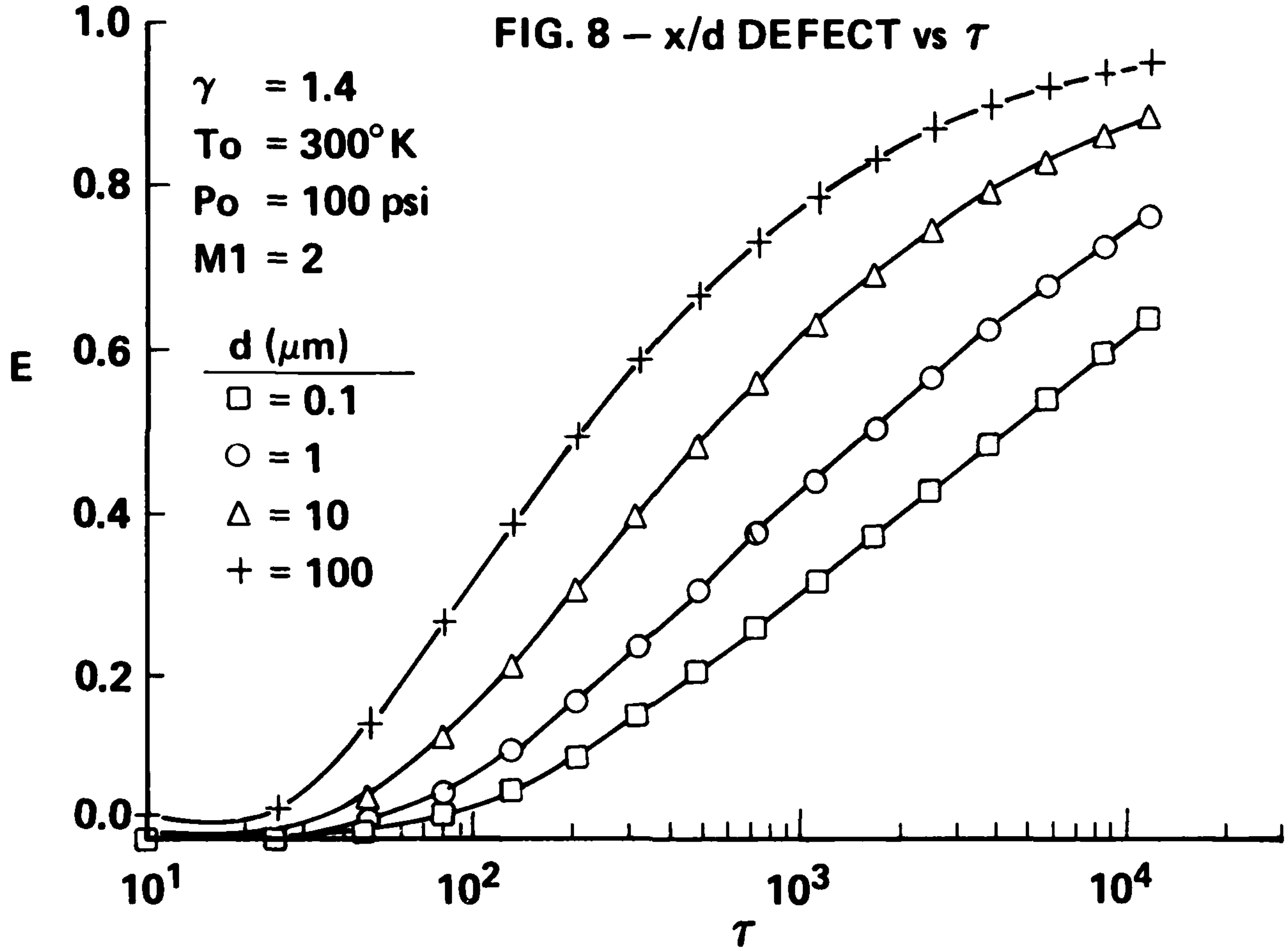


FIG. 8 — x/d DEFECT vs τ

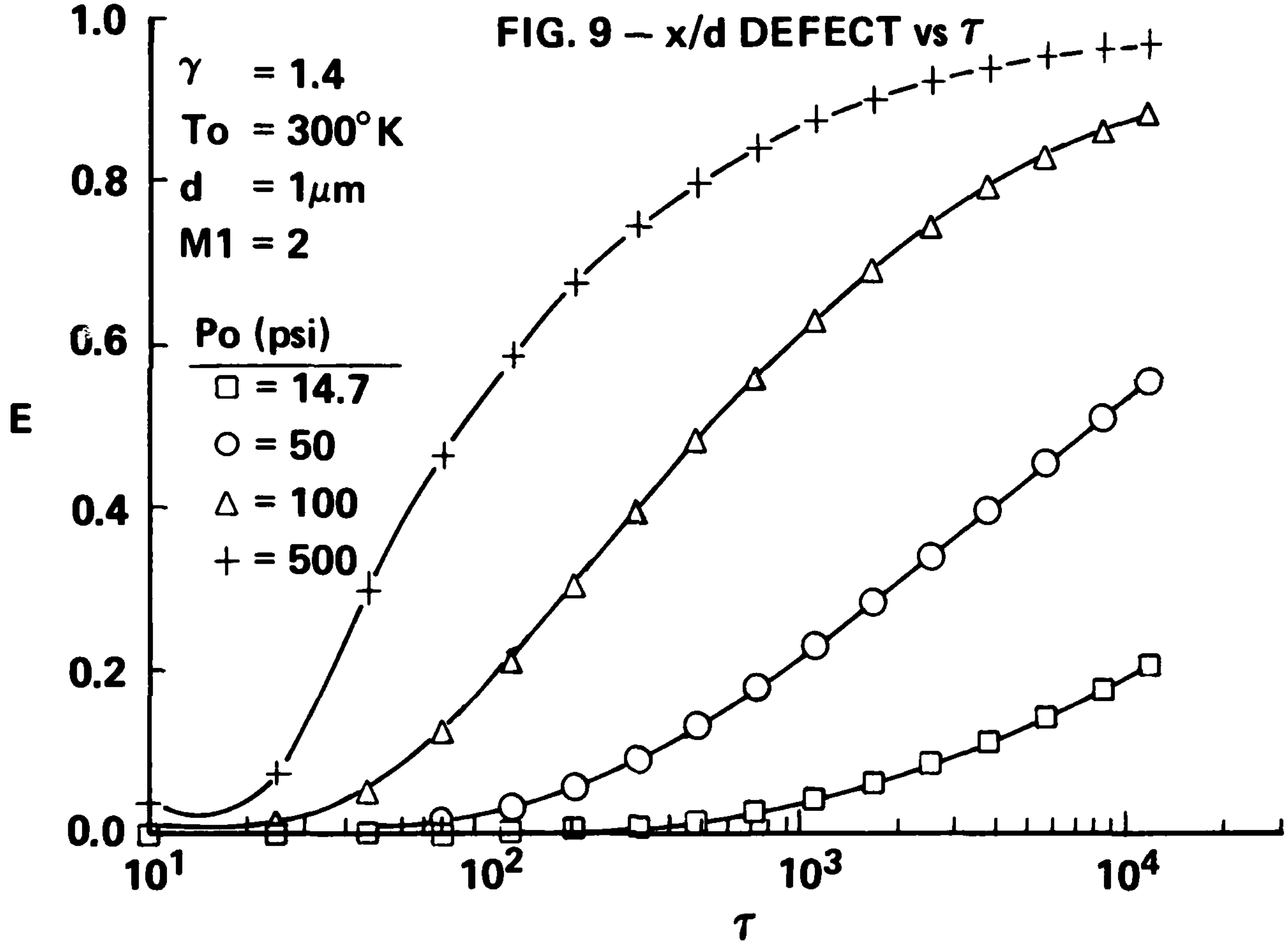


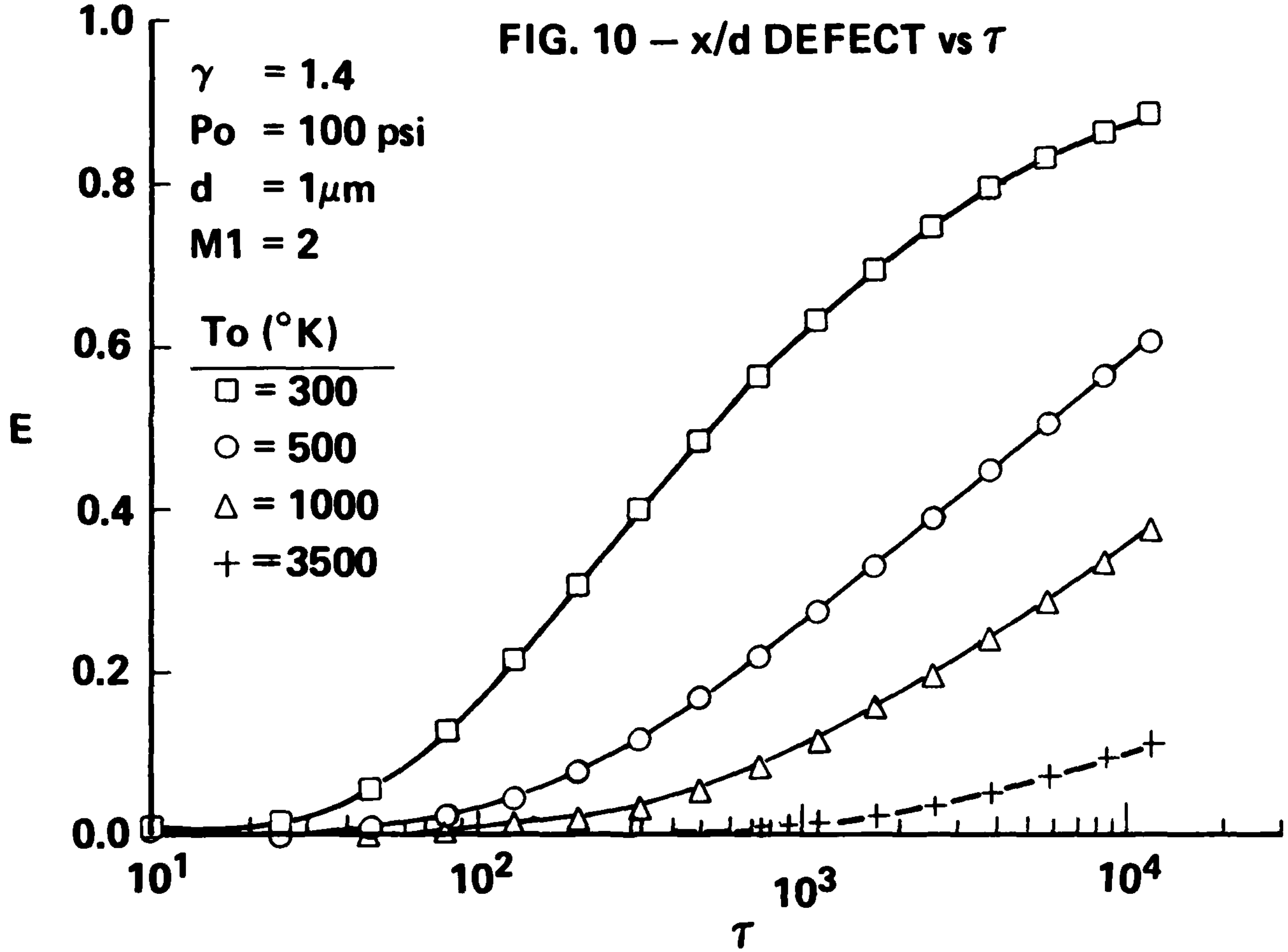
FIG. 9 — x/d DEFECT vs τ

densities, the value of Kn_o from Eq. (9) is reduced and Re_2 increases as indicated in Eq. (14). Thus, the effect of the Basset term increases with increasing stagnation pressures or the particle drag is further reduced and the value of E increases for fixed τ as shown in fig. 9.

6. Effect of Stagnation Temperature

Increases in the stagnation temperature reduce the initial particle Reynolds number Re_2 back of the shock front. Therefore, fig. 10 illustrates a reduction in the particle relaxation defect E with increasing values of T_o.

FIG. 10 — x/d DEFECT vs τ



VIII. REFERENCES

1. Serafini, J. S., "Impingement of Water Droplets on Wedges and Double-Wedge Airfoils at Supersonic Speeds," NACA Report 1159 (1954).

2. Smith, P. W., Stety, G. A. and Delaney, L. J., "Impulse Scaling Predictions," AFRPL-TR-66-297, Air Force Rocket Propulsion Laboratory, Edwards AFB, Edwards, CA (1966).

3. Stein, H. D. and Pfeifer, H. J., "Investigation of the Velocity Relaxation of Micron-Sized Particles in Shock Waves Using Laser Radiation," Applied Optics. 11, 305 (1972).

4. Yanta, W. J., "Measurement of Aerosol Size Distributions with a Laser Doppler Velocimeter," paper no. 73-705, AIAA 6th Fluid and Plasma Dynamics Conference, California (1973).

5. Basset , A. B., "A Treatise on Hydrodynamics," Deighton Bell, Cambridge, England, 1888 (Republished: Dover, New York, 1961).

6. Lewis, J. A. and Gauvin, W. H., "Motion of Particles Entrained in a Plasma Jet," AIChE J. 19, 982 (1973).

7. Torobin, L. B. and Gauvin, W. H., "Fundamental Aspects of a Solids-Gas Flow," Can. J. Chem. Eng. 38, 224 (1959).

8. Ingebo, R. D., "Drag Coefficients for Droplets and Solid Spheres in Clouds Accelerating in Airstreams," NACA T.M. 3762 (1956).

9. Sartor, J. D. and Abbott, C. E., "Prediction and Measurement of the Accelerated Motion of Water Drops in Air," J. Applied Meteor. 14, 232 (1975).

10. Guthrie, R. I. L., Clift, R. and Henein, H., "Contacting Problems Associated with Aluminum and Ferro-Alloy Additions in Steelmaking-Hydrodynamic Aspects," Metall. Trans. B, 6, 321 (1975).

11. Leipmann, H. W. and Roshko, A., Elements of Gasdynamics, John Wiley, New York (1957).

12. Vincenti, W. G. and Kruger, C. H., Jr., Introduction to Physical Gas Dynamics, John Wiley, New York (1965).

13. Jeans, J., An Introduction to the Kinetic Theory of Gases, Cambridge University Press (1940).

14. Forney, L. J., McGregor, W. K. and Girata, P. T., "Particle Sampling with Supersonic Probes: Similitude and Particles Breakup," AEDC-TR-83-26, Arnold Air Force Station, TN (1983).

15. Forney, L. J. and McGregor, W. K., "Scaling Laws for Particle Breakup in Nozzle Generated Shocks," Particulate Science and Technology, 4 (1984).

16. Odar, F. and Hamilton, W. S., "Forces on a Sphere Accelerating in a Viscous Fluid," J. Fluid Mech. 18, 302 (1964).

17. Hjelmfelt, A. T. and Mockros, L. F., "Motion of Discrete Particles in a Turbulent Fluid," Appl. Sci. Res. 16, 149 (1966).

18. Clift, R., Grace, J. R. and Weber, M. E., Bubbles, Drops and Particles, Academic Press (1978).

19. Crowe, C. T., "Drag Coefficient of Particles in a Rocket Nozzle," AIAA J. 5, 1021 (1967).

20. Kellison, G. S., Fundamentals of Numerical Methods, Richard D. Irwin, Inc., Homewood, Illinois (1975).

XI. PUBLICATIONS

1. Forney, L. J., Walker, A. E. and McGregor, W. K., "Effect of the Basset Term on Particle Relaxation Behind Normal Shock Waves," paper #290, Proceedings of the First International Aerosol Conference, Dept. of Mechanical Engineering, University of Minnesota (1984).

2. Walker, A. E., "Effect of the Basset Term on Particle Relaxation Behind Normal Shock Waves," MS Thesis, Dept. of Chemical Engineering, Georgia Institute of Technology, Atlanta, GA (1984).

3. Forney, L. J., Walker, A. E. and McGregor, W. K., "Effect of the Basset Term on Particle Relaxation Behind Normal Shock Waves," Aerosol Science and Technology (to be submitted) 1984.

X. APPENDIX

1. Tabulated Stagnation Conditions

Table 1 - Stagnation density ratio (ρ_p = 1 gm/cc)

Po(psi)	14.7	50	100	500
To(^{0}K)				
300	1.178E-3	4.005E-3	8.011E-3	40.05E-3
500	0.7065E-3	2.403E-3	4.806E-3	24.03E-3
1000	0.3533E-3	1.202E-3	2.403E-3	12.02E-3
3500	0.1009E-3	0.3433E-3	0.6866E-3	03.433E-3

Table 2 - Stagnation Knudsen number (To = 300^0K)

Po(psi)	14.7	50	100	500
d(μm)				
0.1	0.6665	0.19589	0.0979	0.01961
1.0	0.06665	0.019589	9.79E-3	1.961E-3
10.0	6.665E-3	1.9589E-3	9.79E-4	1.961E-4
100.0	6.665E-4	1.9589E-4	9.79E-5	1.961E-5

Table 3 - Stagnation Knudsen number (To = 500^0K)

Po(psi) / d(μm)	14.7	50	100	500
0.1	1.2564	0.3663	0.18316	0.03663
1.0	0.12564	0.03663	0.01832	3.663E-3
10.0	0.012564	3.663E-3	1.8316E-3	3.663E-4
100.0	1.2564E-3	3.663E-4	1.8316E-4	3.663E-5

Table 4 - Stagnation Knudsen number (To = 1000^0K)

Po(psi)	14.7	50	100	500
d(μm)				
0.1	2.74106	0.80567	0.403003	0.080567
1.0	0.274106	0.080567	0.0403003	8.0567E-3
10.0	2.74106E-2	8.0567E-3	4.03003E-3	8.0567E-4
100.0	2.74106E-3	8.0567E-4	4.03003E-4	8.0567E-5

Table 5 - Stagnation Knudsen number (To = 3500^0K)

Po(psi)	14.7	50	100	500
d(μm)				
0.1	10.71826	3.12493	1.56247	0.31249
1.0	1.071826	0.312493	0.156247	3.2149E-2
10.0	0.1071826	3.12493E-2	1.56247E-2	3.1249E-3
100.0	1.071826E-2	3.12493E-3	1.56247E-3	3.12496E-4

2. Numerical Results

RUN 1

M_1 = 1.1 γ = 1.4 $T_0 = 300^0K$ P_0 = 100 psi d = 10 μm

ρ_0/ρ_p = 8.011 E − 3 Kn_o = 9.79 E −4 Re_2 = 158.81 Kn_2 = 1.439 E −3 ρ_2/ρ_p = 5.448 E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	55.395	55.733	1.432	233.815	235.068	0.005
25.000	20.876	22.082	2.435	362.119	364.967	0.008
47.500	7.529	10.276	4.067	433.809	448.961	0.034
81.199	2.347	5.446	6.540	470.453	510.808	0.079
131.699	0.550	3.490	9.343	485.820	564.617	0.140
207.403	0.078	2.595	11.965	490.332	620.729	0.210
320.910	0.005	2.055	14.607	491.065	685.674	0.284
491.121	0.000	1.665	17.547	491.109	763.891	0.357
746.365	0.000	1.363	20.948	491.109	859.577	0.429
1129.171	0.000	1.115	25.086	491.109	976.692	0.497
1703.331	0.000	0.921	29.852	491.109	1121.425	0.562
2565.152	0.000	0.763	35.486	491.109	1300.964	0.623
3858.513	0.000	0.635	42.060	491.109	1524.541	0.678
5798.506	0.000	0.534	49.445	491.109	1805.020	0.728
8705.926	0.000	0.458	57.147	491.109	2162.008	0.773
11990.645	0.000	0.414	62.819	491.109	2519.553	0.805

RUN 2

$M_1 = 1.2$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10\ \mu m$

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 310.54$ $Kn_2 = 1.374$ E −3 $\rho_2/\rho_p = 5.708$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	73.837	74.398	1.237	372.050	374.834	0.007
25.000	24.078	26.376	2.182	531.041	536.571	0.010
47.500	7.990	13.059	3.439	610.485	639.073	0.045
81.199	2.339	7.894	4.929	648.296	722.629	0.103
131.699	0.513	5.677	6.333	663.217	805.669	0.177
207.403	0.066	4.459	7.661	667.307	899.933	0.258
320.910	0.004	3.614	9.080	667.913	1013.014	0.341
491.121	0.000	2.969	10.68	667.938	1151.611	0.420
746.365	0.000	2.456	12.538	667.938	1323.211	0.495
1129.171	0.000	2.027	14.784	667.938	1535.208	0.565
1703.331	0.000	1.687	17.352	667.938	1799.386	0.629
2565.152	0.000	1.407	20.374	667.938	2129.464	0.686
3858.513	0.000	1.177	23.892	667.938	2543.054	0.737
5798.506	0.000	0.993	27.874	667.938	3064.093	0.782
8705.926	0.000	0.849	32.152	667.938	3727.584	0.821
11990.645	0.000	0.762	35.508	667.938	4388.393	0.848

RUN 3

$M_1 = 1.4$ $\gamma = 1.4$ $T_0 = 300^0 K$ $P_0 = 100$ psi $d = 10$ µm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 571.13$ $Kn_2 = 1.325$ E −3 $\rho_2/\rho_p = 5.921$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	87.655	88.427	1.139	532.097	537.541	0.010
25.000	25.754	29.835	2.027	711.150	720.931	0.014
47.500	8.084	16.854	2.897	793.905	844.148	0.060
81.199	2.265	11.560	3.743	831.438	959.384	0.133
131.699	0.474	8.914	4.508	845.619	1085.998	0.221
207.403	0.057	7.208	5.276	849.306	1236.554	0.313
320.910	0.003	5.934	6.119	849.811	1420.934	0.402
491.121	0.000	4.931	7.072	849.830	1649.887	0.485
746.365	0.000	4.116	8.170	849.830	1936.263	0.561
1129.171	0.000	3.426	9.489	849.830	2293.126	0.629
1703.331	0.000	2.872	10.988	849.830	2741.324	0.690
2565.152	0.000	2.410	12.743	849.830	3305.195	0.743
3858.513	0.000	2.027	14.786	849.830	4015.734	0.788
5798.506	0.000	1.713	17.123	849.830	4914.049	0.827
8705.926	0.000	1.460	19.715	849.830	6057.198	0.860
11990.645	0.000	1.300	21.867	849.830	7189.111	0.882

RUN 4

$M_1 = 1.6$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 755.03$ $Kn_2 = 1.355$ E −3 $\rho_2/\rho_p = 5.790$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	96.050	96.906	1.092	624.548	631.743	0.011
25.000	27.697	32.844	1.916	818.709	831.131	0.015
47.500	8.683	19.689	2.619	907.577	970.958	0.065
81.199	2.456	14.019	3.277	948.021	1108.429	0.145
131.699	0.525	10.993	3.878	963.512	1263.488	0.237
207.403	0.066	8.959	4.491	967.647	1449.973	0.333
320.910	0.003	7.415	5.165	968.240	1679.801	0.424
491.121	0.000	6.188	5.925	968.264	1966.542	0.508
746.365	0.000	5.185	6.797	968.264	2326.644	0.584
1129.171	0.000	4.332	7.840	968.264	2777.139	0.651
1703.331	0.000	3.643	9.022	968.264	3344.847	0.711
2565.152	0.000	3.066	10.402	968.264	4061.235	0.762
3858.513	0.000	2.585	12.007	968.264	4966.290	0.805
5798.50#	0.000	2.186	13.851	968.264	6112.443	0.842
8705.926	0.000	1.862	15.915	968.264	7571.421	0.872
11990.645	0.000	1.653	17.661	968.264	9013.513	0.893

RUN 5

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 894.16$ $Kn_2 = 1.596$ E −3 $\rho_2/\rho_p = 4.914$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	117.338	118.246	1.004	736.464	744.606	0.011
25.000	36.013	41.245	1.684	980.233	993.755	0.014
47.500	12.100	23.654	2.333	1099.197	1165.471	0.057
81.199	3.775	16.339	2.956	1157.804	1327.852	0.128
131.699	0.944	12.660	3.512	1183.058	1507.253	0.215
207.403	0.154	10.291	4.062	1191.185	1721.654	0.308
320.910	0.012	8.518	4.658	1192.752	1985.655	0.399
491.121	0.000	7.114	5.326	1192.867	2315.159	0.485
746.365	0.000	5.968	6.089	1192.867	2729.294	0.563
1129.171	0.000	4.996	6.995	1192.867	3248.517	0.633
1703.331	0.000	4.206	8.023	1192.867	3903.613	0.694
2565.152	0.000	3.544	9.222	1192.867	4731.226	0.748
3858.513	0.000	2.990	10.615	1192.867	5777.832	0.794
5798.506	0.000	2.532	12.212	1192.867	7104.601	0.832
8705.926	0.000	2.158	13.997	1192.867	8794.929	0.864
11990.645	0.000	1.918	15.502	1192.867	10466.551	0.886

RUN 6

$M_1 = 2.4$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ µm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 828.63$ $Kn_2 = 2.071$ E −3 $\rho_2/\rho_p = 3.787$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	145.331	146.711	0.931	797.978	804.928	0.009
25.000	50.678	54.604	1.450	1119.027	1130.408	0.010
47.500	19.018	28.124	2.099	1294.839	1346.838	0.039
81.199	6.791	17.341	2.840	1392.290	1528.901	0.089
131.699	2.073	12.542	3.532	1441.415	1711.905	0.158
207.403	0.467	9.926	4.164	*461.471	1920.919	0.239
320.910	0.062	8.132	4.814	1467.095	2174.094	0.325
491.121	0.003	6.754	5.533	1467.944	2487.709	0.410
746.365	0.000	5.646	6.349	1467.966	2879.994	0.490
1129.171	0.000	4.716	7.313	1467.966	3370.798	0.565
1703.331	0.000	3.961	8.413	1467.966	3988.363	0.632
2565.152	0.000	3.330	9.695	1467.966	4766.786	0.692
3858.513	0.000	2.806	11.184	1467.966	5749.404	0.745
5798.506	0.000	2.374	12.882	1467.966	6993.758	0.790
8705.926	0.000	2.028	14.753	1467.966	8579.869	0.829
11990.645	0.000	1.809	16.292	1467.966	10153.622	0.855

RUN 7

$M_1 = 3$ $\gamma = 1.4$ $T_o = 300^0K$ $P_o = 100$ psi $d = 10$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 604.66$ $Kn_2 = 3.330$ E− 3 $\rho_2/\rho_p = 2.355$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	186.716	187.425	0.894	799.620	803.704	0.005
25.000	81.576	83.550	1.191	1258.771	1265.664	0.005
47.500	36.995	41.825	1.675	1567.843	1595.642	0.017
81.199	16.137	22.208	2.422	1774.986	1849.687	0.040
131.699	6.390	13.029	3.432	1905.146	2060.880	0.076
207.403	2.130	8.800	4.530	1977.193	2259.748	0.125
320.910	0.528	6.695	5.553	2009.297	2474.621	0.188
491.121	0.080	5.400	6.552	2019.280	2728.440	0.260
746.365	0.005	4.446	7.640	2021.011	3039.386	0.335
1129.171	0.000	3.680	8.903	2021.099	3424.233	0.410
1703.331	0.000	3.065	10.353	2021.099	3904.006	0.482
2565.152	0.000	2.560	12.045	2021.099	4504.255	0.551
3858.513	0.000	2.145	14.008	2021.099	5257.406	0.616
5798.506	0.000	1.810	16.232	2021.099	6207.088	0.674
8705.926	0.000	1.548	18.621	2021.099	7416.621	0.727
11990.645	0.000	1.390	20.483	2021.099	8622.303	0.766

RUN 8

$M_1 = 4.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ µm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 297.55$ $Kn_2 = 7.742$ E −3 $\rho_2/\rho_p = 1.013$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	189.438	189.723	1.017	588.702	589.690	0.002
25.000	120.755	121.221	1.100	1152.802	1154.789	0.002
47.500	74.768	75.945	1.283	1685.589	1692.458	0.004
81.199	43.992	45.672	1.613	2168.275	2187.439	0.009
131.699	24.096	26.142	2.187	2580.008	2622.972	0.016
207.403	11.997	14.305	3.196	2903.682	2988.164	0.028
320.910	5.238	7.712	4.943	3131.682	3284.414	0.047
491.121	1.886	4.405	7.615	3269.286	3528.818	0.074
746.365	0.503	2.892	10.746	3335.208	3752.089	0.111
1129.171	0.082	2.172	13.706	3357.235	3988.514	0.158
1703.331	0.006	1.737	16.641	3361.441	4265.070	0.212
2565.152	0.000	1.416	19.929	3361.560	4600.745	0.269
3858.513	0.000	1.164	23.745	3361.560	5013.106	0.329
5798.506	0.000	0.965	28.132	3361.560	5523.883	0.391
8705.926	0.000	0.813	32.943	3361.560	6163.601	0.455
11990.645	0.000	0.722	36.724	3361.560	6792.142	0.505

RUN 9

$M_1 = 1.2$ $\gamma = 1.2$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 319.51$ $Kn_2 = 1.385$ E −3 $\rho_2/\rho_p = 5.661$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	75.296	75.866	1.226	380.712	383.574	0.007
25.000	24.547	26.889	2.157	542.783	548.439	0.010
47.500	8.163	13.331	3.391	623.838	652.996	0.045
81.199	2.400	8.071	4.849	662.536	738.365	0.103
131.699	0.531	5.809	6.220	677.897	823.313	0.177
207.403	0.070	4.566	7.517	682.149	919.814	0.258
320.910	0.004	3.702	8.903	682.790	1035.631	0.341
491.121	0.000	3.043	10.467	682.817	1177.637	0.420
746.365	0.000	2.518	12.276	682.817	1353.516	0.496
1129.171	0.000	2.079	14.465	682.817	1570.876	0.565
1703.331	0.000	1.730	16.970	682.817	1841.814	0.629
2565.152	0.000	1.443	19.915	682.817	2180.424	0.687
3858.513	0.000	1.208	23.343	682.817	2604.777	0.738
5798.506	0.000	1.019	27.225	682.817	3139.482	0.783
8705.926	0.000	0.871	31.398	682.817	3820.382	0.821
11990.645	0.000	0.782	34.678	682.817	4497.965	0.848

RUN 10

$M_1 = 1.6$ $\gamma = 1.2$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ µm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 812.21$ $Kn_2 = 1.365$ E −3 $\rho_2/\rho_p = 5.635$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	98.312	99.189	1.081	650.196	657.924	0.012
25.000	28.261	33.724	1.887	848.557	861.769	0.015
47.500	8.869	20.535	2.549	939.250	1006.528	0.067
81.199	2.518	14.749	3.166	980.611	1150.619	0.148
131.699	0.542	11.608	3.732	996.534	1314.109	0.242
207.403	0.069	9.479	4.311	1000.823	1511.243	0.338
320.910	0.004	7.855	4.947	1001.448	1754.562	0.429
491.121	0.000	6.563	5.664	1001.473	2058.497	0.513
746.365	0.000	5.504	6.487	1001.473	2440.583	0.590
1129.171	0.000	4.603	7.468	1001.473	2919.064	0.657
1703.331	0.000	3.874	8.580	1001.473	3522.572	0.716
2565.152	0.000	3.263	9.877	1001.473	4284.732	0.766
3858.513	0.000	2.752	11.386	1001.473	5248.257	0.809
5798.506	0.000	2.329	13.119	1001.473	6469.106	0.845
8705.926	0.000	1.983	15.066	1001.473	8023.308	0.875
11990.645	0.000	1.760	16.718	1001.473	9558.446	0.895

RUN 11

$M_1 = 2.0$ $\gamma = 1.2$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 10$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 958.4$ $Kn_2 = 1.675$ E −3 $\rho_2/\rho_p = 4.68$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	124.522	125.450	0.981	778.354	786.979	0.011
25.000	38.857	44.204	1.621	1038.924	1053.042	0.013
47.500	13.301	25.208	2.243	1168.304	1236.597	0.055
81.199	4.259	17.318	2.844	1233.408	1409.140	0.125
131.699	1.109	13.385	3.380	1262.347	1598.995	0.211
207.403	0.194	10.877	3.904	1272.128	1825.618	0.303
320.910	0.017	9.006	4.471	1274.167	2104.711	0.395
491.121	0.000	7.526	5.104	1274.345	2453.206	0.481
746.365	0.000	6.318	5.827	1274.345	2891.445	0.559
1129.171	0.000	5.293	6.683	1274.345	3441.376	0.630
1703.331	0.000	4.459	7.655	1274.345	4135.640	0.692
2565.152	0.000	3.759	8.787	1274.345	5013.232	0.746
3858.513	0.000	3.173	10.101	1274.345	6123.656	0.792
5798.506	0.000	2.688	11.607	1274.345	7531.829	0.831
8705.926	0.000	2.292	13.293	1274.345	9326.109	0.863
11990.645	0.000	2.036	14.715	1274.345	11100.867	0.885

RUN 12

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 0.1$ μm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 0.979$ E −4 $Re_2 = 8.942$ $Kn_2 = 0.1596$ E −3 $\rho_2/\rho_p = 4.914$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	6.875	6.887	3.965	19.621	19.647	0.001
25.000	4.811	4.864	5.062	41.234	41.335	0.002
47.500	2.976	3.146	7.077	62.631	63.391	0.012
81.199	1.557	1.813	11.208	80.984	83.598	0.031
131.699	0.642	0.933	20.238	93.944	100.089	0.061
207.403	0.186	0.460	39.988	100.865	112.430	0.103
320.910	0.031	0.256	68.410	103.311	121.963	0.153
491.121	0.002	0.176	98.780	103.778	130.802	0.207
746.365	0.000	0.134	128.217	103.812	140.500	0.261
1129.171	0.000	0.106	161.670	103.812	151.831	0.316
1703.331	0.000	0.085	200.545	103.812	165.390	0.372
2565.152	0.000	0.069	247.004	103.812	181.792	0.429
3858.513	0.000	0.056	302.768	103.812	201.760	0.485
5798.506	0.000	0.046	369.306	103.812	226.195	0.541
8705.926	0.000	0.038	446.642	103.812	256.295	0.595
11990.645	0.000	0.033	512.525	103.812	285.281	0.636

RUN 13

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 1.0$ µm

$\rho_0/\rho_p = 8.011$ E − 3 $Kn_o = 9.79$ E −4 $Re_2 = 89.42$ $Kn_2 = 1.596$ E −2 $\rho_2/\rho_p = 4.7$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	39.996	40.179	1.739	148.621	149.213	0.004
25.000	18.044	18.678	2.657	249.668	251.140	0.006
47.500	7.525	9.073	4.311	315.988	324.191	0.025
81.199	2.693	4.529	7.298	354.902	377.728	0.060
131.699	0.741	2.548	11.711	373.713	419.775	0.110
207.403	0.131	1.715	16.471	380.309	458.521	0.171
320.910	0.011	1.304	20.995	381.687	500.430	0.237
491.121	0.000	1.039	25.773	381.813	549.588	0.305
746.365	0.000	0.842	31.234	381.813	608.925	0.373
1129.171	0.000	0.683	37.857	381.813	680.959	0.439
1703.331	0.000	0.560	45.535	381.813	769.327	0.504
2565.152	0.000	0.461	54.656	381.813	878.261	0.565
3858.513	0.000	0.382	65.374	381.813	1013.127	0.623
5798.506	0.000	0.319	77.523	381.813	1181.340	0.677
8705.926	0.000	0.272	90.256	381.813	1394.245	0.726
11990.645	0.000	0.246	99.530	381.813	1606.754	0.762

RUN 14

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 100$ psi $d = 100$ µm

$\rho_0/\rho_p = 8.011$ E - 3 $Kn_o = 9.79$ E -5 $Re_2 = 8941.63$ $Kn_2 = 1.596$ E - 4 $\rho_2/\rho_p = 4.7$ E - 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	193.617	195.435	0.782	2518.246	2596.670	0.030
25.000	47.261	91.497	1.106	2876.110	2989.646	0.038
47.500	14.426	90.709	1.111	3024.909	3526.414	0.142
81.199	4.287	74.739	1.223	3093.144	4216.332	0.266
131.699	1.043	62.637	1.339	3121.467	5074.835	0.385
207.403	0.168	53.217	1.460	3130.391	6161.751	0.492
320.910	0.013	45.546	1.589	3132.089	7551.529	0.585
491.121	0.000	39.154	1.730	3132.195	9340.401	0.665
746.365	0.000	33.705	1.886	3132.195	11651.667	0.731
1129.171	0.000	28.995	2.062	3132.195	14627.832	0.786
1703.331	0.000	25.026	2.255	3132.195	18479.607	0.831
2565.152	0.000	21.598	2.473	3132.195	23466.213	0.867
3858.513	0.000	18.629	2.718	3132.195	29919.855	0.895
5798.506	0.000	16.050	2.998	3132.195	38264.734	0.918
8705.926	0.000	13.815	3.317	3132.195	49042.969	0.936
11990.645	0.000	12.271	3.599	3132.195	59749.801	0.948

RUN 15

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 14.7$ psi $d = 1$ μm

$\rho_0/\rho_p = 1.178$ E − 3 $Kn_o = 6.67$ E − 2 $Re_2 = 13.13$ $Kn_2 = 0.1086$ $\rho_2/\rho_p = 7.226$ E − 4

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	12.486	12.489	2.962	32.017	32.023	0.000
25.000	11.595	11.600	3.092	77.141	77.158	0.000
47.500	10.419	10.436	3.295	138.974	139.052	0.001
81.199	8.946	8.981	3.622	220.331	220.628	0.001
131.699	7.220	7.280	4.167	321.858	322.763	0.003
207.403	5.364	5.455	5.119	439.823	442.173	0.005
320.910	3.570	3.693	6.903	564.468	569.876	0.009
491.121	2.050	2.197	10.579	680.557	691.762	0.016
746.365	0.960	1.118	19.135	771.741	792.812	0.027
1129.171	0.335	0.487	41.249	828.192	864.275	0.042
1703.331	0.075	0.206	94.003	852.973	909.441	0.062
2565.152	0.008	0.112	169.818	859.512	941.054	0.087
3858.513	0.000	0.081	234.646	860.265	971.158	0.114
5798.506	0.000	0.063	297.942	860.265	1005.487	0.144
8705.926	0.000	0.051	371.235	860.265	1046.403	0.178
11990.645	0.000	0.043	438.838	860.265	1084.550	0.207

RUN 16

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 50$ psi $d = 1\ \mu m$

$\rho_0/\rho_p = 4.005\ E-3$ $Kn_o = 1.96\ E-2$ $Re_2 = 44.69$ $Kn_2 = 3.193\ E-2$ $\rho_2/\rho_p = 2.457\ E-3$

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	32.542	32.591	1.939	95.299	95.425	0.001
25.000	21.934	22.079	2.366	195.504	195.871	0.002
47.500	13.469	13.899	3.124	292.385	294.461	0.007
81.199	7.362	7.992	4.563	376.771	383.430	0.017
131.699	3.450	4.190	7.468	441.250	456.693	0.034
207.403	1.302	2.066	13.455	482.504	512.320	0.058
320.910	0.354	1.066	24.174	502.949	553.814	0.092
491.121	0.058	0.662	37.441	509.845	588.678	0.134
746.365	0.004	0.488	49.768	511.135	624.389	0.181
1129.171	0.000	0.384	62.437	511.225	665.482	0.232
1703.331	0.000	0.309	76.878	511.225	714.616	0.285
2565.152	0.000	0.250	93.980	511.225	774.156	0.340
3858.513	0.000	0.204	114.374	511.225	846.809	0.396
5798.506	0.000	0.168	138.479	511.225	935.970	0.454
8705.926	0.000	0.139	165.945	511.225	1046.317	0.511
11990.645	0.000	0.122	188.511	511.225	1153.197	0.557

RUN 17

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 300^0K$ $P_0 = 500$ psi $d = 1$ µm

$\rho_0/\rho_p = 4.005$ E − 2 $Kn_o = 1.961$ E −3 $Re_2 = 446.40$ $Kn_2 = 3.197$ E −3 $\rho_2/\rho_p = 2.456$ E − 2

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	9.992	10.422	4.012	159.954	166.099	0.037
25.000	1.062	10.093	4.104	173.869	187.727	0.074
47.500	0.075	10.432	4.009	175.908	250.502	0.298
81.199	0.002	8.385	4.695	176.071	328.740	0.464
131.699	0.000	6.869	5.448	176.075	423.947	0.585
207.403	0.000	5.690	6.293	176.075	541.574	0.675
320.910	0.000	4.739	7.265	176.075	687.699	0.744
491.121	0.000	3.965	8.381	176.075	871.143	0.798
746.365	0.000	3.283	9.783	176.075	1102.623	0.840
1129.171	0.000	2.741	11.373	176.075	1386.962	0.873
1703.331	0.000	2.319	13.107	176.075	1747.253	0.899
2565.152	0.000	1.962	15.139	176.075	2204.703	0.920
3858.513	0.000	1.651	17.601	176.075	2783.759	0.937
5798.506	0.000	1.382	20.601	176.075	3512.403	0.950
8705.926	0.000	1.154	24.221	176.075	4425.678	0.960
11990.645	0.000	1.001	27.540	176.075	5308.995	0.967

RUN 18

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 500^0K$ $P_0 = 100$ psi $d = 1$ µm

$\rho_0/\rho_p = 4.806$ E − 3 $Kn_o = 1.832$ E −4 $Re_2 = 47.79$ $Kn_2 = 2.986$ E −2 $\rho_2/\rho_o = 2.948$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	32.455	32.518	1.940	98.396	98.562	0.002
25.000	20.381	20.575	2.467	194.888	195.364	0.002
47.500	11.611	12.168	3.421	281.708	284.43	0.010
81.199	5.832	6.607	5.273	351.639	360.137	0.024
131.699	2.462	3.333	9.035	400.392	419.450	0.045
207.403	0.805	1.668	16.292	428.115	463.739	0.077
320.910	0.177	0.948	27.042	439.759	498.640	0.118
491.121	0.021	0.654	38.035	442.829	531.459	0.167
746.365	0.001	0.505	48.469	443.226	567.756	0.219
1129.171	0.000	0.402	60.096	443.235	610.561	0.274
1703.331	0.000	0.325	73.575	443.235	662.160	0.331
2565.152	0.000	0.265	89.606	443.235	724.967	0.389
3858.513	0.000	0.216	108.704	443.235	801.850	0.447
5798.506	0.000	0.178	131.106	443.235	896.506	0.506
8705.926	0.000	0.149	156.107	443.235	1014.220	0.563
11990.645	0.000	0.132	175.890	443.235	1129.207	0.607

RUN 19

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 1000^0K$ $P_0 = 100$ psi $d = 1$ μm

$\rho_0/\rho_p = 2.403$ E − 2 $Kn_o = 4.03$ E −2 $Re_2 = 21.72$ $Kn_2 = 6.569$ E −2 $\rho_2/\rho_p = 1.479$ E − 3

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	18.988	18.999	2.484	50.779	50.805	0.001
25.000	15.745	15.774	2.750	115.627	115.708	0.001
47.500	12.192	12.293	3.192	193.605	194.056	0.002
81.199	8.654	8.833	3.964	280.278	281.927	0.006
131.699	5.493	5.746	5.400	367.653	372.079	0.012
207.403	3.012	3.321	8.301	445.308	455.133	0.022
320.910	1.354	1.687	14.737	503.710	522.751	0.036
491.121	0.457	0.778	29.571	538.573	571.653	0.058
746.365	0.100	0.379	58.002	553.462	605.763	0.086
1129.171	0.011	0.235	91.615	557.317	633.571	0.120
1703.331	0.000	0.176	121.393	557.767	662.369	0.158
2565.152	0.000	0.139	152.581	557.767	695.812	0.198
3858.513	0.000	0.112	189.018	557.767	735.837	0.242
8705.926	0.000	0.074	284.414	557.767	843.241	0.339
11990.645	0.000	0.063	331.853	557.767	899.184	0.380

RUN 20

$M_1 = 2.0$ $\gamma = 1.4$ $T_0 = 3500^0K$ $P_o = 100$ psi $d = 1\ \mu m$

$\rho_0/\rho_p = 6.866\ E-4$ $Kn_o = .1562$ $Re_2 = 5.60$ $Kn_2 = .2547$ $\rho_2/\rho_p = 4.21\ E-4$

Tau	Re	Reb	CD	x/d	xb/d	E
10.000	5.506	5.506	4.017	13.885	13.886	0.000
25.000	5.364	5.364	4.088	34.264	34.266	0.000
47.500	5.160	5.162	4.196	63.858	63.867	0.000
81.199	4.874	4.878	4.363	106.113	106.146	0.000
131.699	4.483	4.490	4.626	165.131	165.240	0.001
207.403	3.968	3.982	5.049	244.968	245.281	0.001
320.910	3.328	3.350	5.755	348.141	348.974	0.002
491.121	2.587	2.621	6.993	473.179	475.224	0.004
746.365	1.810	1.856	9.333	611.778	616.427	0.008
1129.171	1.091	1.149	14.249	747.411	757.145	0.013
1703.331	0.532	0.596	26.154	858.850	877.468	0.021
2565.152	0.190	0.251	59.871	930.127	962.388	0.034
3858.513	0.042	0.094	157.436	961.761	1012.262	0.050
5798.506	0.005	0.044	332.834	969.950	1042.169	0.069
8705.926	0.000	0.030	484.405	970.775	1067.813	0.091
11990.645	0.000	0.025	591.448	970.775	1089.985	0.109

3. Numerical Code

```
      INTEGER UPP,FM,TOP,DPRIN
      REAL KN,MACH,NUMST,DRAG,NUO,NU,MAC,MAC2,KNO
C
C
C
      REAL TAU(120001),DEVB(120001),REB(120001)
      PI=3.1415926
      DH=.48
      PR=.7
C
C     INITIAL CONDITIONS
C
      GA=1.4
      MAC=2.
      KNO=.000979
      RORP=.008011
C
      GM2=(GA-1.)/2.
      GP2=(GA+1.)/2.
      MAC2=MAC**2
      ROR2=(1.+GM2*MAC2)**(GA/(GA-1.))
     1/(GP2*MAC2)
      KN=KNO*ROR2
      F3=(MAC2-1.)/(GM2**.5*(1.+GM2*MAC2)**.5*
     1 (-1.+(GA/GM2)*MAC2)**.5)
      REIO=(PI*GA/2.)**.5*1./KN*F3
      PGP=RORP*KNO/KN
      B=4.5*PGP
      TR=GP2**2/GM2*MAC2/
     1((1.+GM2*MAC2)*(GA/GM2*MAC2-1.))
      TRB=TR
      CD = 24./REIO
      WRITE(6,*) REIO,KN,B,F3
C
C     INITIAL VARIABLES
C
      TAU(1) = 0.
      TAUMAX=12000.
      NUMST=120000.
      DELTA = TAUMAX/NUMST
      DEVO=-B*CD*REIO**2/24.
      DEVB(1)=-B*CD*REIO**2/24.
      REB(1) = REIO
      INC=3
      IST1=200
      DELO=160
      TOP=20
      DPRIN=100
      K=1
      J=1
      I=1
      N1=1
      SUME=0.
      SUMB=0.
```

```
      REF = REIO
      REO = REIO
      ERR=.00000001
C
      WRITE(6,400)
C
C   FOURTH ORDER RUNGE KUTTA FOR RE
C
      DO 100 I=2,NUMST+1
      ROO=REIO/REO
      IF(ROO .LE. ERR) GO TO 500
      RO=REIO
      R1=RO+DEVO*DELTA/2.*B
      CD=DRAG(R1,KN,GA,TR)
      D1=(-CD/24.)*R1**2
      R2=RO+D1*DELTA/2.*B
      CD=DRAG(R2,KN,GA,TR)
      D2=(-CD/24.)*R2**2
      R3=RO+D2*DELTA*B
      CD=DRAG(R3,KN,GA,TR)
      D3=(-CD/24.)*R3**2
      DD=(1./6.)*DEVO+(1./3.)*D1+(1./3.)*D2+(1./6.)*D3
      REI=RO+DELTA*DD*B
      CD=DRAG(REI,KN,GA,TR)
      DEVN=(-CD/24.)*REI**2
C
C     CALCULATE PARTICLE TEMPERATURE
C
      MACH=KN*REI*(2./(PI*GA))**.5
      NUO=2.+.459*REI**.55*PR**.33
      RPM=MACH/(REI*PR)
      NU=NUO/(1.+3.42*NUO*RPM)
      DTR=3./2.*NU/PR*PGP*.9*(GA-1.)/2.*MACH**2.
      TR=TR+DTR*DELTA
C
C    CALCULATE PARTICLE RELAXATION DISTANCE
C
      TSUM=.5*(REI+REIO)*DELTA*.25
      SUME=SUME+TSUM
C
C
  500 CONTINUE
      TAU(I)=TAU(I-1)+DELTA
C
C
C
C     ANALYTICAL EXPRESSION FOR B=9.
C
      IF(K.EQ.200000)THEN
      CONTINUE
      ELSE
      GO TO 370
      ENDIF
      SQB=SQRT(1.-4./B)
```

```
      AL=.5*B*(1.+SQB)
      BE=.5*B*(1.-SQB)
      EXB=BE*TAU(I)**.5
      EXA=AL*TAU(I)**.5
      EXB2=EXB**2
      EXA2=EXA**2
      IF(EXB.LE.5.)GO TO 310
      SXB=0.
      DO 320 M=1,5
      FM1=FM(M)
      TXB=-1.**M*FM1/(2.*EXB2)**M
      SXB=SXB+TXB
 320  CONTINUE
      ERB=1./(SQRT(PI)*EXB)*(1.+SXB)
      GO TO 330
 310  ERB=EXP(EXB2)*ERFC(EXB)
 330  IF(EXA.LE.5.)GO TO 350
      SXA=0.
      DO 340 M=1,5
      FM1=FM(M)
      TXA=-1.**M*FM1/(2.*EXA2)**M
      SXA=SXA+TXA
 340  CONTINUE
      ERA=1./(SQRT(PI)*EXA)*(1.+SXA)
      GO TO 360
 350  ERA=EXP(EXA2)*ERFC(EXA)
 360  REF=AL/(AL-BE)*ERB+BE/(BE-AL)*ERA
      REF=REO*REF
 370  CONTINUE
C
C
C         THE FOLLOWING LOOP NUMERICALLY EVALUATES THE BASSET
C         CONTRIBUTION.
        IF(I.EQ.2)GO TO 150
C
C
C        TRAPEZOIDAL RULE UP TO IST1 OR DELO IF I .GE. IST1

        UPP=I-2
         IF(I .GE. IST1)UPP=DELO
       SO=0.
        DO 630 J=2,UPP
        TBA=DEVB(J)/(TAU(I)-TAU(J))**.5*2.
        SO=SO+TBA
 630    CONTINUE
       DOO=DEVB(1)/TAU(I)**.5
        DNO=DEVB(UPP+1)/(TAU(I)-TAU(UPP+1))**.5
        BASUP1=DELTA/2.*(DOO+DNO+SO)
        IF(I.GE.IST1) GO TO 615
        GO TO 150
C
C        SIMPSONS RULE WITH INC
C
 615    IF(N1.EQ.2*IST1) THEN
```

```
      INC=INC+2
      N1=1
      ELSE
      N1=N1+1
      ENDIF
      S1=0.
      DO 635 J=UPP+INC+1,I-1-INC-TOP,2*INC
      L=J
      TBA=DEVB(J)/(TAU(I)-TAU(J))**.5*4.
      S1=S1+TBA
 635  CONTINUE
      S2=0.
      DO 645 J=UPP+2*INC+1,I-1-2*INC-TOP,2*INC
      TBA=DEVB(J)/(TAU(I)-TAU(J))**.5*2.
      S2=S2+TBA
 645  CONTINUE
      DOO=DEVB(UPP+1)/(TAU(I)-TAU(UPP+1))**.5
      DNO=DEVB(L+INC)/(TAU(I)-TAU(L+INC))**.5
      BASLINC=(S1+S2 +DNO+DOO)*INC*DELTA/3.+BASUP1
      S3=0.
C
C     FINISHES WITH MODIFIED INTEGRATION
C
      IF(L+INC.EQ.I-1) GO TO 660
      T=TAU(I)
      DO 650 J=L+INC,I-2
      ADEV=DEVB(J)+DEVB(J+1)
      T1=TAU(J)
      T2=TAU(J+1)
      TBA=ADEV*((T-T1)**.5-(T-T2)**.5)
      S3=S3+TBA
 650  CONTINUE
 660  BAS1=S3+BASLINC
      GO TO 150
C
C
C     COMPUTES NEW DEVB(I),REB(I)
C  USES PREDICTOR CORRECTOR FOR REB
C
 150  SQ=1./SQRT(PI)*B
      SQ1=1./SQRT(PI)*DELTA**.5*B
      DRO=DEVB(I-1)
      RO=REB(I-1)
      BAO=BAS1
      BA1=SQ*BAO+DRO*SQ1*2.
      RO1=RO+DRO*DELTA*B
      CD=DRAG(RO1,KN,GA,TRB)
      D1=-CD/24.*RO1**2-DH*BA1
      BA11=SQ*BAO+SQ1*(DRO+D1)
      R1=RO+.5*(D1+DRO)*DELTA*B
      CD=DRAG(R1,KN,GA,TRB)
      D2=-CD/24.*R1**2-DH*BA11
      R2=RO+.5*(D2+DRO)*DELTA*B
      CD=DRAG(R2,KN,GA,TRB)
```

```
      BA2=SQ*BAO+SQ1*(DRO+D2)
      D3=-CD/24.*R2**2-DH*BA2
      R3=RO+.5*(D3+DRO)*DELTA*B
      CD=DRAG(R3,KN,GA,TRB)
      DEVB(I)=D3
      REB(I)=R3
      CD1=CD
C
C     CALCULATE PARTICLE TEMPERATURE
C
      MACH=KN*REB(I)*(2./(PI*GA))**.5
      NUO=2.+.459*REB(I)**.55*PR**.33
      RPM=MACH/(REB(I)*PR)
      NU=NUO/(1.+3.42*NUO*RPM)
      DTR=3./2.*NU/PR*PGP*.9*(GA-1.)/2.*MACH**2
      TRB=TRB+DTR*DELTA
C
C    CALCULATE PARTICLE RELAXATION DISTANCE
C
      TSUM=.5*(REB(I)+REB(I-1))*DELTA*.25
      SUMB=SUMB+TSUM
C
C
       ROO=REB(I)/REO
       IF(ROO.LE.ERR)GO TO 510
C
C
C
        DELREB =( REI - REB(I))/REB(I)
        DSUM=1.-SUME/SUMB
        IF (K .EQ. DPRIN) THEN
           WRITE (6,200) TAU(I), REI, REB(I), CD1, REF,
     1     DELREB , SUME,SUMB,DSUM,TR,TRB
           DPRIN=3*DPRIN*1/2
           K = 1
       ELSE
           K = K+1
      ENDIF
C
      DEVO=DEVN
      REIO=REI
      IF(I.NE.NUMST) GO TO 800
      WRITE(6,200) TAU(I),REI,REB(I),CD1,REF,DELREB,SUME,
     1 SUMB,DSUM,TR,TRB
      GO TO 100
  800 CONTINUE
  100 CONTINUE
 400  FORMAT(8X,'TAU',8X,'RE',8X,'REB',8X,'CD',8X,'REF',8X,
     1 'DELREB',6X, 'SUME',6X,'SUMB',6X,'DSUM',6X,'TR',6X,'TRB')
 200    FORMAT(1X,11(F10.3,1X) )
 510  CONTINUE
      STOP
      END
C
```

```
C
      REAL FUNCTION DRAG(REB,KN,GA,TR)
      REAL MACH,KN
      PI=3.1415926
      MACH=KN*REB*(2./(PI*GA))**.5
       CDP = 24./REB*(1.+.158*REB**.6667)
       H=(2.3+1.7*(TR**.5))-2.3*TANH(1.17*ALOG10(MACH))
      GN=10.**(1.25*(1.+TANH(0.77*ALOG10(REB)-1.92)))
      DRAG=(CDP-2.)*EXP(-3.07*(GA**.5)*MACH/REB*GN)
     1+H/(MACH*(GA**.5))*EXP(-REB/(2.*MACH))+2.
      RETURN
      END
C
C
      INTEGER FUNCTION FM(M)
        FM=1
        DO 700 N=1,M
        FM=FM*(2*N-1)
 700    CONTINUE
        RETURN
        END
```